Search for the High-Rank Symmetries in Subatomic Physics

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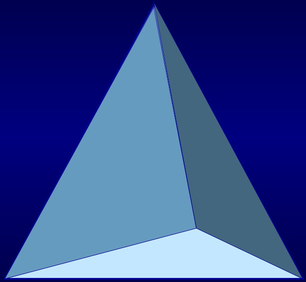
A Brief Remainder: Platonic Tetrahedral Symmetry

Let us recall one of the magic forms introduced long time by Plato. The implied symmetry leads to the *tetrahedral group* denoted T_d

A tetrahedron has four equal walls. Its shape is invariant with respect to 24 symmetry elements. Tetrahedron is *not* invariant with respect to the inversion. Of course nuclei cannot be represented by a sharp-edge pyramid



... but rather in a form of a regular spherical harmonic expansion:

$$\mathcal{R}(\vartheta,\varphi) = R_0\, c(\{\alpha\})[1 + \sum_{\lambda}^{\lambda_{max}} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda,\mu}\, Y_{\lambda,\mu}(\vartheta,\varphi)]$$

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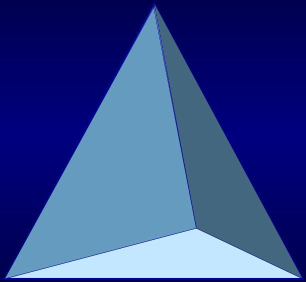

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A Basis for Tetrahedral Symmetry

Only special combinations of spherical harmonics may form a basis for surfaces with tetrahedral symmetry and only odd-order:

Three Lowest Order Solutions: Rank $\leftrightarrow$ Multipolarity λ

$$\lambda = 3: \quad \alpha_{3,\pm 2} \equiv t_3$$

$\lambda = 5:$ no solution possible

$$\lambda = 7: \quad \alpha_{7,\pm 2} \equiv t_7; \quad \alpha_{7,\pm 6} \equiv -\sqrt{\tfrac{11}{13}} \cdot t_7$$

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Nuclear Tetrahedral Shapes - 3D Examples

Illustrations below show the tetrahedral-symmetric surfaces at three increasing values of rank $\lambda = 3$ deformations t_1: 0.1, 0.2 and 0.3

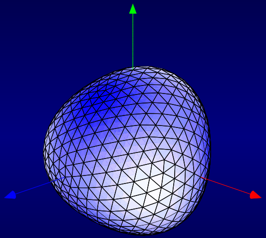


Figure: $t_3 = 0.1$

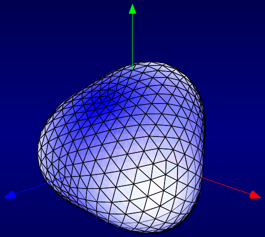


Figure: $t_3 = 0.2$

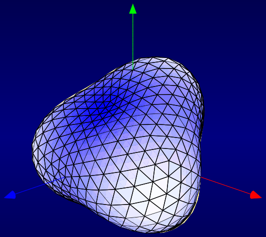


Figure: $t_3 = 0.3$

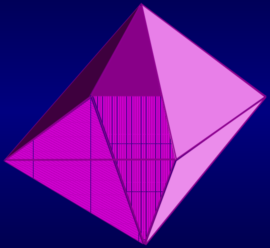
Observations:

- There are infinitely many tetrahedral-symmetric surfaces
- Nuclear 'pyramids' do not resemble pyramids!

A Brief Remainder [2]: Platonic Octahedral Symmetry

Let us recall one of the magic forms introduced long time by Plato. The implied symmetry leads to the *octahedral group* denoted O_h

An octahedron has 8 equal walls. Its shape is invariant with respect to 48 symmetry elements that include inversion. However, the nuclear surface cannot be represented in the form of a diamond $\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$



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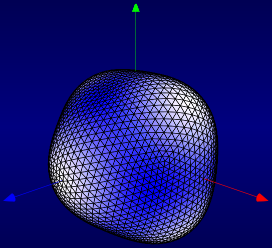

Figure: $o_4 = 0.1$

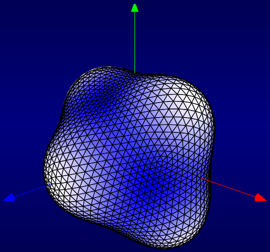

Figure: $o_4 = 0.2$

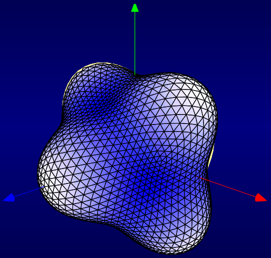

Figure: $o_4 = 0.2$

Recall: $\alpha_{40} \equiv o_4; \quad \alpha_{4,\pm 4} \equiv \pm\sqrt{\frac{5}{14}} \cdot o_4$

Example: Results with the HFB Solutions in RE

The HFB results for tetrahedral solutions in light Rare-Earth nuclei

$$\boxed{\alpha_{4,0} \equiv o_4, \quad \alpha_{4,\pm 4} \equiv -\sqrt{5/14}\, o_4}$$

Z	N	ΔE (MeV)	Q_{32} $(b^{3/2})$	Q_{40} (b^2)	Q_{44} (b^2)	$Q_{40} \times \sqrt{\frac{5}{14}}$ (b^2)
64	86	-1.387	0.941817	-0.227371	$+0.135878$	-0.135880
64	90	-3.413	1.394656	-0.428250	$+0.255929$	-0.255928
64	92	-3.972	0.000000	-0.447215	$+0.267263$	-0.267262
62	86	-0.125	0.487392	-0.086941	$+0.051954$	-0.051957
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We Proceed to Construct Theoretical Predictions

- Assume that Q_2-moments of the special configurations vanish
- We associate the negative-parity bands with $Q_{3\mu}$ type shapes
- Let us consider 4 cases: All deformations equal to zero, except for $\alpha_{30} \neq 0$, $\alpha_{31} \neq 0$, $\alpha_{32} \neq 0$ and $\alpha_{33} \neq 0$, one at the time
- In what follows we consider two types of comparison:
  - Multipole moments within uniform density distribution
  - Realistic total energy calculations on the $(\alpha_{3\mu_1} - \alpha_{3\mu_2})$-plane

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Multipole Moments as Functionals of the Density

- For small deformations we use Taylor expansion:

$$Q_{\lambda\mu}(\alpha) \approx Q_{\lambda\mu}\Big|_{\alpha=0} + Q'_{\lambda\mu}\Big|_{\alpha=0}\Delta\alpha + \frac{1}{2}Q''_{\lambda\mu}\Big|_{\alpha=0}\Delta\alpha\Delta\alpha$$

- We set $\lambda = 2$, $\mu = 0$ and $\lambda_1 = \lambda_2 = 3$ and obtain

$$\begin{aligned}
\alpha_{30}: \quad Q_{20} &= 15/(2\sqrt{5\pi}) \cdot \alpha_{30}^2 \cdot \rho_0\, R_0^5 \\
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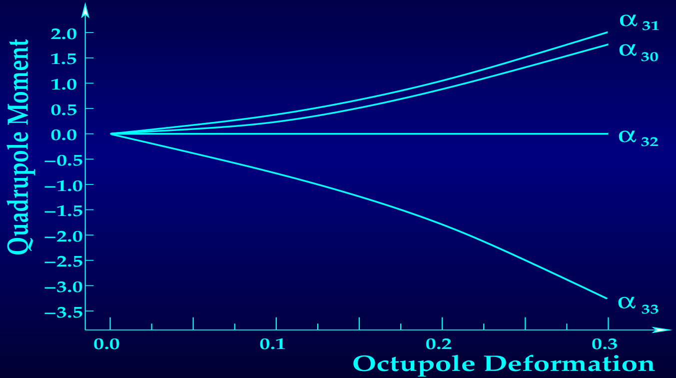
Similar Results with the Woods-Saxon Hamiltonian

Microscopic Multipole Moments: $Q_{20}(\alpha_{3\mu}) = \int \Psi^*_{WS}(\tau)\hat{Q}_{20}\Psi_{WS}(\tau)d\tau$



Observe that $Q_{20}(\alpha_{32})$ vanishes identically in the W.S. case as well

We can now formulate the experimental criteria!

The So-Called ‘Smoking Guns’

- Tetrahedral nuclei are deformed → they produce collective rotation
- The lowest order T_d−symmetry is $Y_{3\pm2}$ → negative parity bands
- At the exact symmetry limit Q_2 moments must vanish! Therefore:
- There must exist negative-parity bands without E2 transitions !!!

We suggest looking for the collective negative parity bands without ‘rotational’ (E2) transitions. The question : Where?

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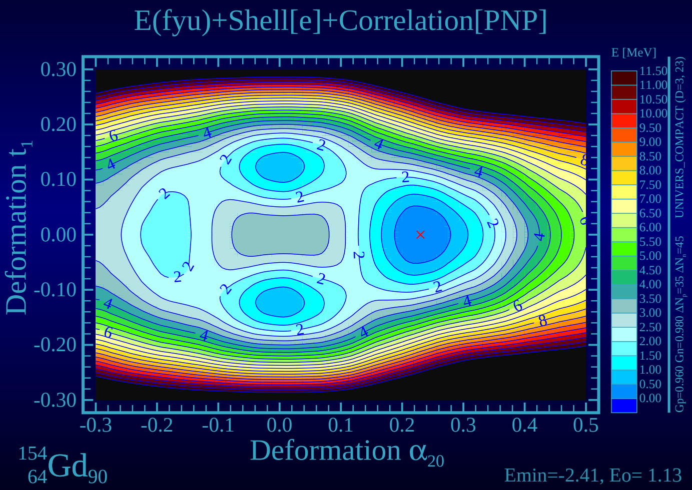
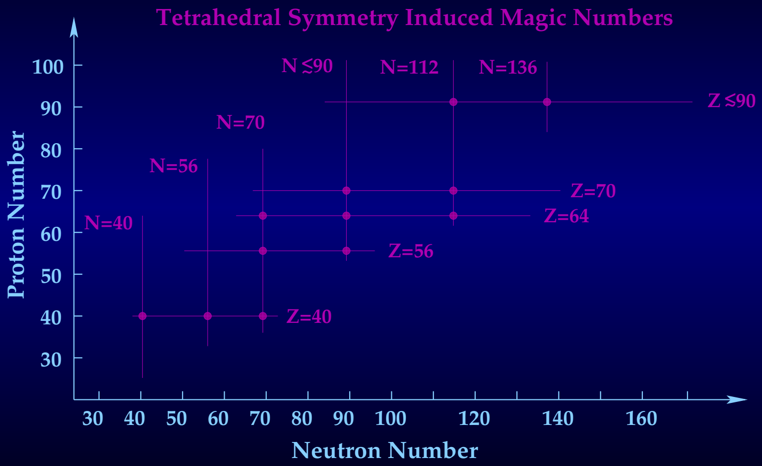
Survey of Doubly-Magic Tetrahedral Nuclei



Tetrahedral Stability; Tetrahedral Magic Numbers



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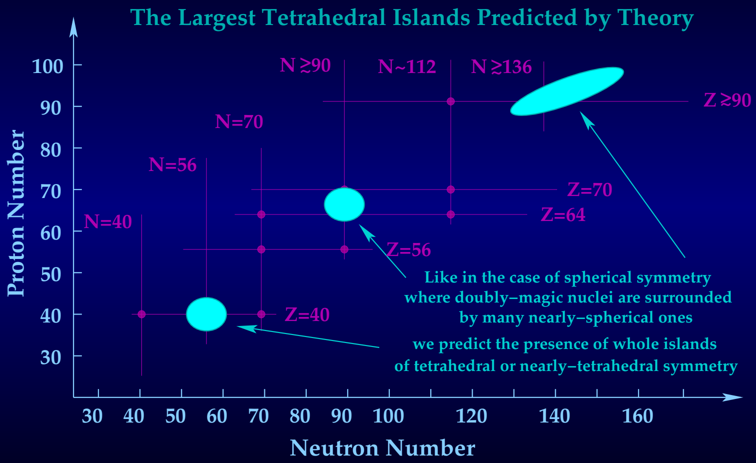


Tetrahedral/Octahedral Shapes Have No Q_2-Moments

At the exact tetrahedral symmetry the quadrupole moments vanish

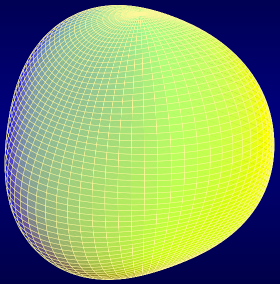
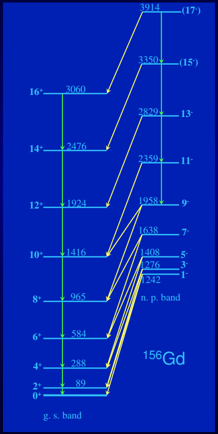
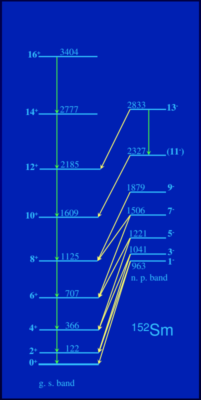


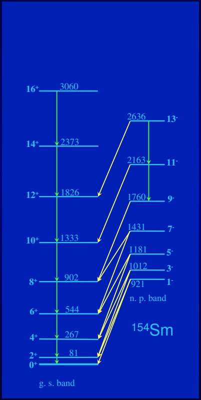
Figure: *Equilibrium shape* $t_1 = 0.15$.

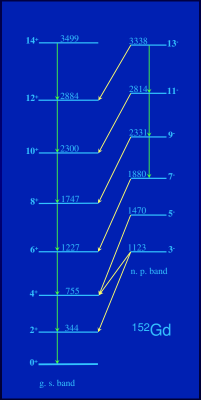


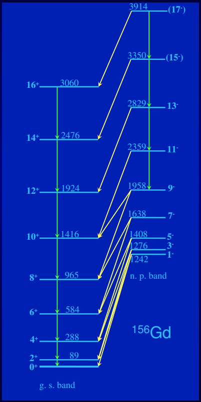
Evidence for Vanishing E2 Transitions in Rare Earths

Despite numerous tries nobody has ever succeed in observing E2's









The bands are identified thanks to the E1 transitions to the GSBs

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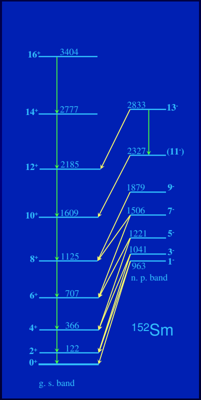

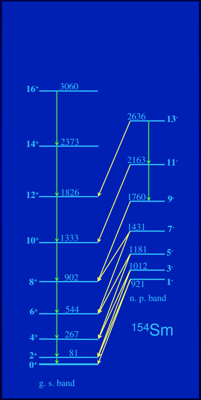

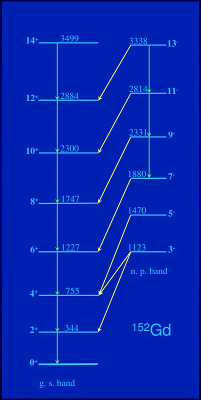

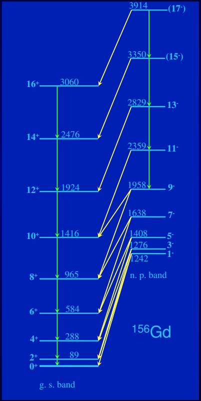

The bands are identified thanks to the E1 transitions to the GSBs

Evidence for Vanishing E2 Transitions in Actinides

The E2 transitions not seen in $^{230-234}$U, while seen in ^{236}U; the experimental conditions (γ and ec) are the same or comparable

Evidence for Vanishing E2 Transitions in Actinides

Comparison: Observe the 'tetrahedral' band patterns in ^{232}U in both the negative and positive parities!

Nuclear Tetrahedral Shapes - Unexpected Features !

The Schrödinger equation is solved for the collective quadrupole-octupole Hamiltonian (A. Dobrowolski et al., Int. J. Mod. Phys. E20 (2011) 500):

$$\mathcal{H}_{coll.} = \frac{-\hbar^2}{2\sqrt{|B|}} \sum_{\{i,j\}=2}^{3} \frac{\partial}{\partial \alpha_{i\mu}} \sqrt{|B|} \left[B(\alpha_{2\mu}, \alpha_{3\mu'})^{-1} \right]_{ij} \frac{\partial}{\partial \alpha_{j\mu'}} + V(\alpha_{2\mu}, \alpha_{3\mu'})$$

The intertia tensor $B_{ij}(\alpha_{20}, \alpha_{3\mu})$ is determined within the Inglis-Belayev cranking approximation.

Due to the use of curvilinear coordinates the element of volume integration contains the term $\sqrt{\det(B)}$ which will have important consequences since the probability-density is essentially given by

$$P \sim |\Psi|^2 \cdot \sqrt{\det(B)}$$

Nuclear Tetrahedral Shapes - Unexpected Features !

Potential energy surface for ^{156}Dy as function of $\{\alpha_{20}, \alpha_{32}\}$ deformation parameters (left) and square root of the determinant (right)

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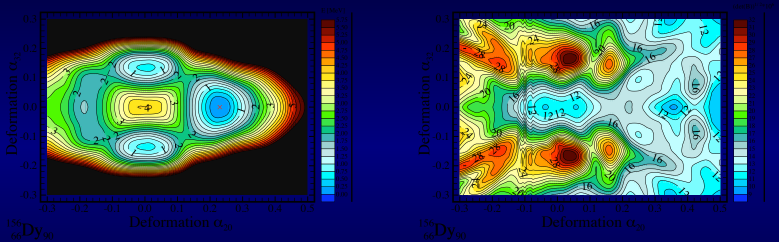
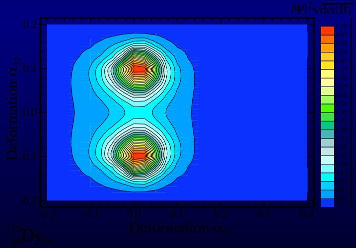


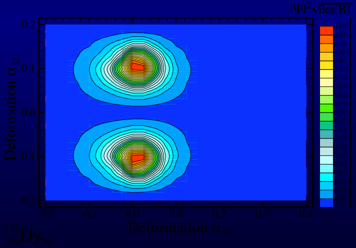
Observations:

- Note strong maxima in $\sqrt{\det(B)}$ at the tetrahedral minima!
- This leads to the enhancement of the probability of occupying the tetrahedral symmetry configurations through dynamical (collective-mass) effects.

Nuclear Tetrahedral Shapes - Unexpected Features !

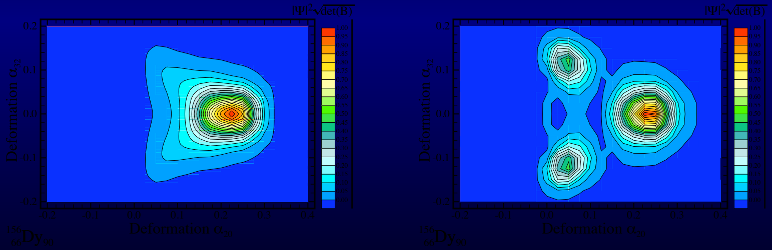
Probability density distributions $|\Psi|^2 \cdot \sqrt{\det(B)}$ *for the ground-state and the first excited state, the latter happening to be of the negative parity and lying only about 160 keV about the ground-state.*





Nuclear Tetrahedral Shapes - Unexpected Features !

The same, but for the third and fourth excited collective-vibration states in ^{156}Dy; the third excitation (left) lies 670 keV above the ground-state, the fourth (right) at 760 keV above.



Note that in the last case the integrated probability of being tetrahedral is 60% and quadrupole is only 40%.

Summary and Conclusions

• *In the historical approach, a "static" view of the problem has lead to the conclusion that an exact tetrahedral symmetry implies a vanishing of the quadrupole moments and transitions.*

• *In the more refined description, there is an enhancement of finding the tetrahedral configuration in the system through the appearence of the integration volume term which is given by the square-root of the determinant of the inertia tensor.*

• *According to the more advanced formalism, tetrahedral configurations coexist with quadrupole ones. This may lead to strong B(E2) values associated with tetrahedral structures !*

• *The recent article by M. Jentschel et al. Phys. Rev. Lett. 104 (2010) 222502 seems to confirm such an expectation !!*